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$$\therefore 2\triangle\cos\gamma=\begin{vmatrix}1, & 1, & 1\\ x_2, & x_3, & x_4\\ y_2, & y_3, & y_4\end{vmatrix}=\begin{vmatrix}x_3-x_2, & x_4-x_2\\ y_3-y_2, & y_4-y_2\end{vmatrix}.$$

Similarly,

$$2\triangle\cos\beta=\begin{vmatrix}z_3-z_2, & z_4-z_2\\ x_3-x_2, & x_4-x_2\end{vmatrix},\quad 2\triangle\cos\alpha=\begin{vmatrix}y_3-y_2, & y_4-y_2\\ z_3-z_2, & z_4-z_2\end{vmatrix}.$$

Also, $-p=(x_1-x_2)\cos\alpha+(y_1-y_2)\cos\beta+(z_1-z_2)\cos\gamma.$

$$\therefore -6V=-2\triangle p=2\triangle\cos\alpha(x_1-x_2)+2\triangle\cos\beta(y_1-y_2)+2\triangle\cos\gamma(z_1-z_2)$$

$$=(x_1-x_2)\begin{vmatrix}y_3-y_2, & y_4-y_2\\ z_3-z_2, & z_4-z_2\end{vmatrix}+(y_1-y_2)\begin{vmatrix}z_3-z_2, & z_4-z_2\\ x_3-x_2, & x_4-x_2\end{vmatrix}$$

$$+(z_1-z_2)\begin{vmatrix}x_3-x_2, & x_4-x_2\\ y_3-y_2, & y_4-y_2\end{vmatrix}$$

$$=\begin{vmatrix}x_1-x_2, & x_3-x_2, & x_4-x_2\\ y_1-y_2, & y_3-y_2, & y_4-y_2\\ z_1-z_2, & z_3-z_2, & z_4-z_2\end{vmatrix}=-\begin{vmatrix}1, & 1, & 1, & 1\\ x_1-x_2, & 0, & x_3-x_2, & x_4-x_2\\ y_1-y_2, & 0, & y_3-y_2, & y_4-y_2\\ z_1-z_2, & 0, & z_3-z_2, & z_4-z_2\end{vmatrix}$$

Multiply the first row of this last determinant by x_2, y_2, z_2 and add to the second, third, fourth, respectively, we get

$$V=\tfrac{1}{6}\begin{vmatrix}1, & 1, & 1, & 1\\ x_1, & x_2,, & x_3, & x_4\\ y_1, & y_2,, & y_3, & y_4\\ z_1, & z_2,, & z_3, & z_4\end{vmatrix}$$

Also solved by S. G. Barton, and V. M. Spunar.

366. Proposed by G. I. HOPKINS, A. M., Professor of Mathematics and Astronomy, Manchester, N. H.

Construct a triangle, having given the base, vertical angle, and difference of altitude and difference of other two sides.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Mo.

Let A be the given vertical angle, a the given side, and d=given difference. Then $h-(b-c)=d$, or $b-c=h-d$...(1).

We have also $b^2+c^2-2bc\cos A=a^2$, which by means of (1) changes into $4bc\sin^2\frac{1}{2}A=a^2-(h-d)^2$, whence $bc=\dfrac{a^2-(h-d)^2}{4\sin^2\frac{1}{2}a}$; but $bc\sin A=ah$.

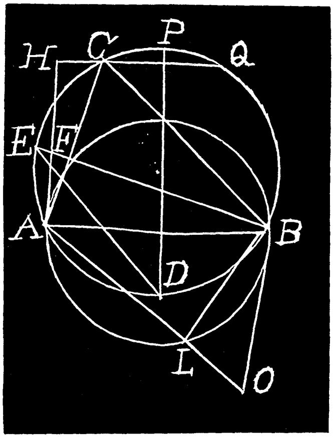
$\therefore\ \dfrac{a^2-(h-d)^2}{4\sin^2\frac{1}{2}a}=\dfrac{an}{\sin A}$, and thus we finally get the quadratic equation in h, $h^2-2(d-a\tan\frac{1}{2}A)h=a^2-d^2$. This quadratic is easily constructed, and after having h the construction of the triangles is very easy.

Solved similarly by V. M. Spunar.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.**

Let $AB=a$ be the base, ACB the given vertical angle, $p=z-(q-y)$ where z=altitude; x, y the remaining sides, respectively. On AB describe the segment of the circle containing the given vertical angle. Draw the diameter PD perpendicular to AB. Take $DE=AB$ and draw BE. Erect AF perpendicular to AB and also describe a circle on AB as diameter. Draw a line $AL=p$ meeting this circumference in L. Draw LB. Take $LO=AL-AF$ where LO is AL produced. Draw BO. Produce AF to H making $AH=BO+LO$, and draw HCQ parallel to AB. Then ACB is the triangle required. For $xy\sin C=az$, $x-y=z-p$, $a^2=x^2+y^2-2xy\cos C$. Hence $(z-p)^2=a^2-2xy(1-\cos C)=a^2-2az\tan\frac{1}{2}C$.

$\therefore z=p-a\tan\frac{1}{2}C+\sqrt{[(p-a\tan\frac{1}{2}C)^2+a^2-p^2]}$.



$\angle ABE=\frac{1}{2}C$, $AF=a\tan\frac{1}{2}C$, $AL=p$, $BL=\sqrt{[a^2-p^2]}$.
$LO=p-a\tan\frac{1}{2}C$, $BO=\sqrt{[(p-a\tan\frac{1}{2}C)^2=a^2-p^2]}$.
$\therefore AH=z$ and ABC is the required triangle.

CALCULUS.

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292. **Proposed by J. SCHEFFER, A. M., Hagerstown, Md.**

Integrate the partial differential equation, $x^2\dfrac{\partial z}{\partial x}+y^2\dfrac{\partial z}{\partial y}=axy$.

Solution by PROF. W. W. BEMAN, University of Michigan.

This is problem 12, page 297, of Johnson's *Differential Equations.*

$$\frac{dx}{x^2}=\frac{dy}{y^2}=\frac{dz}{dxy}.$$

From the first two fractions,

$$\frac{1}{x}-\frac{1}{y}=i. \quad \therefore x=\frac{y}{1+cy}.$$